2023A5 WIND TURBINE TOWER

**Level 1:** If the first section of an 80-meter wind turbine tower has a diameter of 4.5 meters and the third tower section has a taper of 1°, what is the diameter at the top of the third tower section? Assume each tower section is equal in length.

This problem can be solved using geometry. We can use a right triangle to solve for the diameter at the top of the third tower section. We can set up this right triangle in a couple of ways. Two examples are shown below; Option A is on the left and Option B is on the right.

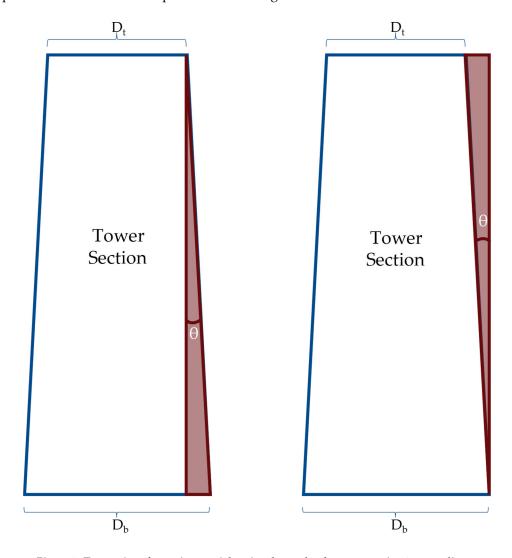


Figure 2: Two options for setting up right triangle to solve for tower section 3 upper diameter

Each of these options involve the same mathematical setup, it just comes down to how you prefer to visualize the problem.

To solve for the diameter of the top of the third tower section, we need to solve for the short side of our right triangle, we'll call it x, using the equation below.

$$\tan \theta = \frac{opposite}{adjacent} = \frac{x}{section \ length}$$

We know our angle and section length, so we can solve for the base of the right triangle, x.

$$\tan 1 = \frac{x}{20 \ m}$$

$$x = 0.349 m$$

Now that we've solved for x, we need to multiply it by 2 to account for both sides tapering on the tower section, which gives 0.698 meters. This means the diameter at the top of the third tower section is 3.802 meters.

**Level 2:** For the same 80-meter wind turbine tower in the Level 1 question, solve for the angle of taper for the fourth tower section. Assume that the ratio of bottom to top diameter of the fourth tower section is 0.78.

To solve this question, we can use the same math as the Level 1 question, but we can use a system of equations to solve for the top diameter of the fourth tower section.

$$an \theta = \frac{opposite}{adjacent} = \frac{x}{section \ length}$$

$$x = \frac{D_b - D_t}{2}$$

$$D_t = 0.78D_b$$

We can combine all of these equations to solve for the angle of taper.

$$\tan \theta = \frac{\frac{1}{2}(D_b - D_t)}{section \ length} = \frac{\frac{1}{2}(0.22D_b)}{20 \ m}$$
$$\tan \theta = 0.0055D_b$$

Plugging in our answer for the top of the third tower section diameter, which is equal to the bottom of the fourth tower diameter, we can solve for the angle of taper for the fourth tower section.

$$\theta = \tan^{-1}(0.0055 * 3.802)$$
$$\theta = 1.198^{\circ}$$